Random vibration protection of a double-chamber submerged jet impingement cooling system: A continuous model

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**Abstract**

High-powered embedded computing equipment using air transport rack (ATR) form-factors is playing an ever-increasing role in aerospace applications. High power and wattage of the electronics and processors require large heat dissipation. Thus more sophisticated, more efficient, and sometimes exotic thermal cooling systems like loop heat pipes or jet impingement systems are demanded. Despite their better thermal performance, these systems are usually more susceptible to mechanically harsh environment where they are deployed. Random vibration is one of the primary excitation sources in aerospace environments where highly efficient cooling systems are used. In this article, a multiple degree of freedom model of an isolated double-chamber jet impingement cooling system is developed, and its response to random vibration is analyzed and compared to that of a traditional single degree of freedom model. Vibration isolation system properties are optimized to minimize the vibration of the internal components of the cooling system, while the whole system enclosure is confined within an allowed rattle space.

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1. Introduction

Electronic components are becoming increasingly miniaturized and integrated. This allows for computers and electronic equipment with dense printed circuit boards (PCB) to become more compact in dimensions [13]. The result is an increasing heat flux at both the component and the circuit board levels. Conventional methods for cooling are not sufficiently effective for these systems. Research into more advanced methods such as liquid-cooling and spray-cooling is being done. These methods can provide higher heat dissipation power and can be used in rugged environments if they are ruggedized accordingly. The objective of this paper is to develop a good model of vibration isolation for a liquid-cooling system.

In this paper a double-chamber submerged jet impingement cooling system is used to provide 2 kW of heat dissipation for an electronic equipment. The CAD model of the impingement system is illustrated in Fig. 1. In this system, water is pumped into the chamber housing, and is then pushed through tiny nozzles (with a diameter of 1 mm) of the nozzle plates at two sides of the chamber housing. Thus, the water is accelerated and impinges with higher velocity on the impingement plates at two sides of the cooling system. Impinged water on the impingement plates thus removes heat from the heating sources that are mounted back to back on the impingement plates (heating sources are not shown in Fig. 1). A thermal diagram of the cooling system is depicted in Fig. 2.

The cooling chamber shown in Fig. 1 along with other components is accommodated within an Air Transport Rack (ATR) chassis as shown in Fig. 3. It should be noted that the Heat Rejection Unit (HRU) where the heated water is cooled down is not placed inside the chassis. The whole chassis with the included components is subjected to severe random vibration in three directions (2 lateral and 1 vertical) from the bottom face (base) of the chassis. The main objective of this research is to protect the cooling system shown in Figs. 1 and 3 against the random base excitations. The aim of vibration protection is to prevent any loss in the thermal performance of the cooling system and any mechanical failures. This will be done by designing an optimum passive vibration isolation system.

The traditional optimal design for vibration isolation from random vibration is based on a trade-off choice of damping and stiffness properties of isolation mounts. The traditional design is focused primarily on optimizing the dynamic response of the entire enclosure or chassis, subject to limitations imposed on their maximum vibration travel (rattle space) [15]. However, some devices include lightly-damped components that are extremely responsive over a wide frequency range. These components are not considered...
Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>σ</td>
<td>standard deviation</td>
</tr>
<tr>
<td>S</td>
<td>power spectral density</td>
</tr>
<tr>
<td>T</td>
<td>transfer function</td>
</tr>
<tr>
<td>k_c</td>
<td>stiffness of the vibration isolation system</td>
</tr>
<tr>
<td>c_c</td>
<td>damping coefficient of the vibration isolation system</td>
</tr>
<tr>
<td>m_c</td>
<td>chassis mass (including the components inside)</td>
</tr>
<tr>
<td>ω_c</td>
<td>undamped natural frequency of the vibration isolation system</td>
</tr>
<tr>
<td>ζ_c</td>
<td>damping ratio of the vibration isolation system</td>
</tr>
<tr>
<td>x_c</td>
<td>chassis displacement</td>
</tr>
<tr>
<td>x_b</td>
<td>base displacement</td>
</tr>
<tr>
<td>z</td>
<td>chassis displacement relative to the base</td>
</tr>
<tr>
<td>AF</td>
<td>attenuation factor</td>
</tr>
<tr>
<td>w_d</td>
<td>absolute displacement of the plate model</td>
</tr>
<tr>
<td>w_b</td>
<td>boundary displacement of the plate model</td>
</tr>
<tr>
<td>w</td>
<td>deflection of the plate model</td>
</tr>
<tr>
<td>a</td>
<td>length of the plate model</td>
</tr>
<tr>
<td>b</td>
<td>width of the plate model</td>
</tr>
<tr>
<td>ρ</td>
<td>aerial density of the plate model</td>
</tr>
<tr>
<td>c</td>
<td>viscous damping coefficient of the plate model</td>
</tr>
<tr>
<td>D</td>
<td>flexural rigidity of the plate model</td>
</tr>
<tr>
<td>ψ_mn</td>
<td>mth eigenfunction of the plate model</td>
</tr>
<tr>
<td>q_mn</td>
<td>mth time-varying modal amplitude of the plate model</td>
</tr>
<tr>
<td>ω_mn</td>
<td>mth undamped natural frequency of the plate model</td>
</tr>
<tr>
<td>f_mn</td>
<td>mth frequency bandwidth of the mth mode of the plate model</td>
</tr>
<tr>
<td>β_mn</td>
<td>damping ratio of the mth mode of the plate model</td>
</tr>
<tr>
<td>m_mn</td>
<td>modal mass of the mth mode of the plate model</td>
</tr>
<tr>
<td>c_mn</td>
<td>modal damping of the mth mode of the plate model</td>
</tr>
<tr>
<td>E[⋯]</td>
<td>expected (mean) value</td>
</tr>
</tbody>
</table>

in the traditional vibration isolation system design. Consequently, traditionally designed vibration isolation systems ignoring these internal components are insufficient for maintaining a fail-safe vibration environment for the system.

Although the traditional design may fulfill the shock or vibration requirements in some applications [1,2,5,10,12,21], it does not always result in an optimum design. Therefore, the conventional method of vibration isolation system design has been challenged in the last decade in some areas like electronics, cryogenic coolers and hard disk drives. Veprik and Babitsky added a secondary degree of freedom for a PCB to the model of an isolated rigid electronic box [16]. They tried to optimize the vibration isolator’s parameters for a harmonic excitation to minimize the PCB’s absolute acceleration or relative deflection, subject to an overall rattle space. Later, Veprik used the same model to optimize the isolator’s properties this time for random vibration input [15].

Veprik and his colleagues designed an isolation system for an infrared equipment to minimize the vibration-induced line-of-sight jitter which was caused by the relative deflection of the infrared sensor and the optic system [17]. They added a second degree of freedom for the cold finger of the cryogenic cooler to the main SDOF model of the system to improve the model as a more realistic 2DOF system. Later on, they used this 2DOF model which included the sensitive cantilevered cold finger to design optimum compliant snubbers for impact protection [18]. There has been some research work conducted to enhance vibration protection for workers working with percussion machines taking into account internal components of these machines for a better design of the protection system [3,4,14].

The authors previously designed an integrated linear–nonlinear vibration isolation system for the ATR chassis mentioned above catering for base random vibration input in presence and absence of g-loading [7]. However, their model ignored the presence of the internal components of the cooling system that could be excited by the input vibration. Hence, in their more recent research study, the authors have developed a 3DOF model of the cooling chamber (a different design of the cooling system from the one studied in
this article), and designed an optimum isolation system for shock and random base excitations [8]. In that study, they added two uncoupled degrees of freedom to the conventional SDOF model. These two added degrees of freedom represented the two nozzle plates in the system. Each nozzle plate had only one mode that could be excited by the excitation frequency range considered in that study. The impingement cooling system used in that study was a single-chamber design, capable of dissipating 1 kW of heat; hence, there needed to be two sets of those cooling chambers to dissipate a total power of 2 kW (1 kW per heating source).

However, in the current study a double-chamber impingement system is designed and employed that was shown earlier in Fig. 1. The double-chamber design was chosen for two reasons: 1) it is a space saving design as it enables the sharing of the pressurized feeding chamber and 2) the diametrically opposed jets will enable any changes in the heat transfer performance due to the orientation effect (present in aerial applications) to be picked up. Unlike the previous design, the heating sources used for this cooling system are quite light with almost negligible flexural rigidity as compared to the impingement plate. Furthermore, the nozzle plate is thinner with larger unsupported area as compared to the previous design. Consequently, the impingement and nozzle plates will both have modes (more than one according to the modal analysis presented later in this article) that could be excited by the excitation frequency range considered. As a result, the SDOF model is no longer useful for this design and a more complex mathematical model is needed.

2. Mathematical modeling

2.1. Traditional SDOF model of the isolated system

Traditional SDOF model of the isolated system is shown in Fig. 4 where the chassis and the included cooling chamber and the two micro pumps are modeled as a single rigid object with mass m. Isolator stiffness and damping are designated by $k_c$ and $c_c$. Using random vibration theory [11] and transmissibility functions for an SDOF model, variance of absolute acceleration of the chassis and random base excitations [8]. In that study, they added two uncoupled degrees of freedom to the conventional SDOF model. These two added degrees of freedom represented the two nozzle plates in the system. Each nozzle plate had only one mode that could be excited by the excitation frequency range considered in that study. The impingement cooling system used in that study was a single-chamber design, capable of dissipating 1 kW of heat; hence, there needed to be two sets of those cooling chambers to dissipate a total power of 2 kW (1 kW per heating source).

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$$\sigma_z^2 = \int_0^{\infty} S_R(j\omega) \left| T_{abs}(j\omega) \right|^2 d\omega$$

$$= \int_0^{\infty} S_R(j\omega) \left| \frac{(\omega^2 + 2j\omega\omega_c\zeta_c)}{(\omega^2 - \omega^2 + 2j\omega\omega_c\zeta_c)} \right|^2 d\omega,$$  \hspace{1cm} (1)

and

$$\sigma_x^2 = \int_0^{\infty} \frac{S_R(j\omega)}{\omega^4} \left| T_{rel}(j\omega) \right|^2 d\omega$$

$$= \int_0^{\infty} \frac{S_R(j\omega)}{\left| \omega^2 - \omega^2 + 2j\omega\omega_c\zeta_c \right|^2} d\omega,$$  \hspace{1cm} (2)

where $z$ is the chassis relative displacement ($z = x_c - x_b$), and $S_R$ is the base one-sided Power Spectral Density (PSD). Conventional notation is used for undamped natural frequency ($\omega_c$) and damping ratio ($\zeta_c$) as below,

$$\omega_c = \sqrt{\frac{k_c}{m_c}} \quad \text{and} \quad \zeta_c = \frac{c_c}{2\sqrt{k_c m_c}}.$$  \hspace{1cm} (3)

The base excitation is Gaussian random excitation with zero mean value. Thus, as the system is linear, the response will also be Gaussian with a zero mean. Consequently, it allows us to use the $3\sigma$ rule to estimate the maximum vibration travel of the chassis as,

$$z_{\text{peak}} = 3\sigma_x.$$  \hspace{1cm} (4)

Fatigue damage accumulation of sensitive components depends mainly on the transmitted absolute acceleration [6]; hence, to assess and quantify the quality of vibration isolation a parameter called attenuation factor (AF) is introduced and defined as the root mean square (rms) value of the absolute acceleration of the hard mounted chassis (that is equal to the rms value of the input base acceleration) divided by that of the isolated chassis as follows,

$$AF = \frac{\sigma_x}{\sigma_z}.$$  \hspace{1cm} (5)

2.2. Proposed continuous model of the isolated system

The chassis is relatively thick and is made of steel, hence stiff enough to be considered rigid for the frequency range of the excitation considered in this article, i.e. 20 Hz to 2000 Hz. There are two l-drive IEG micro pumps in the chassis that apply quite small harmonic forces to the chassis that are considered negligible. However, the three components of the impingement cooling system mentioned earlier could be flexible to the excitation normal to the impingement and nozzle plates.

2.2.1. Finite element modal analysis

In order to check if the three components of the cooling system namely, chamber housing, nozzle plate and impingement plate have any modes that could be excited by the random base vibration ranging from 20 Hz to 2000 Hz, modal analysis of the components is conducted in ANSYS 13.

For the three components, SOLID186-20node is used for the element type. This element is a higher-order, 3D, and 20-node solid element that exhibits quadratic displacement behavior. The element is defined by 20 nodes having three degrees of freedom per node: translation in the x, y and z directions. SOLID186 Homogeneous Structural Solid is well suited for modeling irregular meshes (such as those produced by various CAD/CAM systems). Hexahedral meshing is used for mesh generation. Furthermore, Block Lanczos method is chosen as the mode extraction method. This is a very efficient algorithm to perform a modal analysis for large models. It is a fast and robust algorithm and used for most applications as the default solver.

Encastre constraint is used for the boundary conditions of the three components of the cooling system. Encastre boundary condition is a constraint on all displacements and rotations at a node. The chamber housing is constrained at its two sides where it is locked to the chassis by the wedge locks (Fig. 3). The nozzle and impingement plates are constrained at the inner surface of all the 10 screw holes (Fig. 1).

The chamber housing, just like the previous design of the chamber [8], is stiff enough to be considered rigid. The nozzle plate with dimensions of 177 mm × 127 mm × 4.5 mm has 21 × 14 nozzles with diameters of 1 mm. The tiny nozzles require a very fine
Table 1
Comparison between natural frequencies of the quarter nozzle plates with and without nozzles.

<table>
<thead>
<tr>
<th>Vibration mode number</th>
<th>Natural frequency (Hz) With nozzles</th>
<th>Natural frequency (Hz) Without nozzles</th>
<th>Discrepancy percentage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1840</td>
<td>1848</td>
<td>0.43</td>
</tr>
<tr>
<td>2</td>
<td>4476</td>
<td>4501</td>
<td>0.56</td>
</tr>
<tr>
<td>3</td>
<td>7205</td>
<td>7248</td>
<td>0.60</td>
</tr>
<tr>
<td>4</td>
<td>7922</td>
<td>7953</td>
<td>0.39</td>
</tr>
<tr>
<td>5</td>
<td>8290</td>
<td>8339</td>
<td>0.59</td>
</tr>
</tbody>
</table>

Table 2
The first six natural frequencies of the nozzle and impingement plates.

<table>
<thead>
<tr>
<th>Vibration mode number</th>
<th>Natural frequency (Hz) Nozzle plate</th>
<th>Natural frequency (Hz) Impingement plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1855</td>
<td>1900</td>
</tr>
<tr>
<td>2</td>
<td>3002</td>
<td>2947</td>
</tr>
<tr>
<td>3</td>
<td>4223</td>
<td>4216</td>
</tr>
<tr>
<td>4</td>
<td>4510</td>
<td>4342</td>
</tr>
<tr>
<td>5</td>
<td>4598</td>
<td>4755</td>
</tr>
<tr>
<td>6</td>
<td>5465</td>
<td>5471</td>
</tr>
</tbody>
</table>

Just like the nozzle plate, the impingement plate is also fully constrained at its screw holes and its undamped modal analysis is conducted in ANSYS. Table 2 shows the first six natural frequencies of the nozzle plate (without nozzle holes), and the impingement plate. Figs. 5 and 6 show the corresponding first six mode shapes of the nozzle and impingement plates, respectively. A rule of thumb in modal analysis states that all frequencies below one and half times the maximum excitation frequency should be considered for dynamic analysis. Therefore, there are two modes in both the nozzle and the impingement plates that should be considered for the dynamic analysis. The number of these modes could be higher if the plates are thinner, or they have a larger unsupported area, or they are made of a material with smaller flexural rigidity.

2.2.2. Mathematical modeling and formulation

According to the Finite Element (FE) modal analysis discussed in the preceding section, when the chassis is excited in the lateral direction and normal to the plane of the nozzle and impingement plates, these two plates will have more than one mode to be excited. Therefore, a more comprehensive model catering for flexibility of the two plates should be developed for a more accurate dynamic analysis and a better vibration isolation system design. It should be noted that SDOF model could still be used for the excitation in vertical direction or in the lateral direction that is parallel to the two plates. This is because the in-plane frequencies of the plates are much larger than out of plane frequencies, and hence they could be considered rigid in those directions.

Fig. 7 shows schematic of a new model taking into account the flexibility of the two mentioned plates when the chassis is excited in the lateral direction normal to these plates. In this model, the chassis along with the two pumps and the chamber housing are modeled as rigid. The impingement and nozzle plates are rectangular and quite thin, i.e. the thickness-to-length ratio is small. They have almost a uniform thickness throughout their surface, and they are restricted along their perimeter by a few screws. Therefore, they are modeled as thin rectangular plates with uniform thickness clamped at their four edges. The impingement plate is modeled as a thin plate with dimensions 233 mm × 160 mm × 7 mm, and the nozzle plate is modeled as a thin plate with dimensions 177 mm × 127 mm × 4.5 mm. Both plates are made of aluminum and clamped at their four edges. With these dimensions the first two analytical natural frequencies of the clamped impingement plate are 1825 Hz and 2889 Hz as compared to 1900 Hz and 2947 Hz obtained from the FE modal analysis of the actual impingement plate with actual boundary conditions. The discrepancy in natural frequencies is less than 4% for the impingement plate. The first two analytical natural frequencies of the clamped nozzle plate are 1901 Hz and 3111 Hz as compared to 1855 Hz and 3002 Hz obtained from the FE modal analysis of the actual nozzle plate with actual boundary conditions. The discrepancy between natural frequencies of the actual nozzle plate and its simplified thin plate model is also less than 4%. Hence, the thin rectangular plate model with uniform thickness adequately resembles and models the actual plates.

In this model, the heating sources as shown in Fig. 3 are so light with relatively small flexural rigidity as compared to the impingement plate; hence, they are neglected in the dynamic modeling. However, even if a heating source with considerable flexural rigidity is used, this model can still be used with a modification in the flexural rigidity of the impingement plate. Furthermore, the fluid structure interaction is also neglected for this analysis. By pumping the water into the chamber housing a constant pressure difference is kept across the two sides of the nozzle plate resulting in a small deflection of the plates. Since this deflection is quite small it will not affect the dynamic response of the nozzle plates significantly. Nonetheless, the water flow will add some damping to the system that is not considered in this analysis.

The ratio of the mass of the nozzle and impingement plates to that of the chassis (including the chamber housing and the pumps) is less than 2%. As a result, it can be assumed that dynamics of the impingement and nozzle plates will not affect dynamics of the chassis significantly. Therefore, the chassis can be considered as an SDOF system, and the response of this SDOF model to the input base excitation is considered as the input excitation for the plates. To this end, random vibration response of clamped thin rectangular plates to boundary excitations should be investigated.

Fig. 8 shows a thin rectangular plate with general boundary conditions subjected to boundary excitation. a and b are length and width of the plate, and \( w(x, y, t) \) are displacements of the boundary and deflection (displacement relative to the boundary) of an arbitrary point on the plate, respectively. The governing dynamic equation of the plate is given by

\[
\rho \frac{\partial^2 w}{\partial t^2} + c \frac{\partial w}{\partial t} + D \nabla^4 w = 0, \tag{6}
\]

where \( w \) is absolute displacement of the plate and is given by

\[
w(x, y, t) = w_b(t) + w(x, y, t). \tag{7}\]

In Eq. (6) \( \rho \) and \( c \) are areal density and viscous damping coefficient per unit area, respectively, and \( D \) is the flexural rigidity of
Fig. 5. The first six mode shapes of the nozzle plate (without nozzles) with the full constraints at screw holes.

Fig. 6. The first six mode shapes of the impingement plate.
the plate. By substituting Eq. (7) into Eq. (6) and rearranging the equation, one obtains

$$\frac{\partial^2 w}{\partial t^2} + c \frac{\partial w}{\partial t} + D \nabla^4 w = -\rho \ddot{w}_h(t). \tag{8}$$

Now making use of mode summation technique, deflection of the plate is assumed to have the form

$$w(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \psi_{mn}(x) \phi_{mn}(y) q_{mn}(t), \tag{9}$$

where \(\psi_{mn}\) and \(q_{mn}\) are \(mn\)th eigenfunction and time varying amplitude function.

By substituting Eq. (9) into Eq. (8) and then multiplying both sides of the resultant equation by \(\psi_{kl}(x,y)\) and integrating over the plate’s surface, in view of orthogonality of the eigenfunctions, one can arrive at the following equation,

$$\ddot{q}_{mn}(t) + \rho_\omega^2 q_{mn}(t) + \omega_{mn}^2 q_{mn}(t) = -\frac{1}{\rho \omega} \int_0^\infty \psi_{mn}(x) \rho \ddot{w}_h(t) \, dx \, dy, \tag{10}$$

where \(\omega_{mn}\) is undamped natural frequency of the \(mn\)th mode of the plate and \(\rho_\omega\) is frequency bandwidth of the \(mn\)th mode and is defined as,

$$\rho_\omega = \frac{c_{mn}}{\omega_{mn}} = 2\xi_{mn}\omega_{mn}. \tag{11}$$

In the last equation, \(\xi_{mn}\) is the modal damping ratio of the \(mn\)th mode, and \(m_{mn}\) and \(c_{mn}\) are called modal mass and modal damping, respectively (\(c_{mn} = (c/m)m_{mn}\)).

The frequency response function of the plate at point \((x, y)\), \(H_w(x, y, \omega)\) to boundary excitation can be calculated by applying a unit harmonic input as \(w_h(t) = e^{i\omega t}\). By applying the unit harmonic input to Eq. (10) one obtains

$$\ddot{q}_{mn}(t) + \rho_\omega^2 q_{mn}(t) + \omega_{mn}^2 q_{mn}(t) = \frac{\rho \omega^2 I_{mn}}{m_{mn}} e^{i\omega t}, \tag{12}$$

where,

$$I_{mn} = \int \psi_{mn}(x, y) \, dx \, dy. \tag{13}$$

The solution of Eq. (12) may be written as

$$q_{mn}(t) = \frac{\rho \omega^2}{m_{mn}(\omega^2 + \beta_{mn} \omega + \alpha_{mn}^2)} I_{mn} e^{i\omega t}$$

$$= H_{mn}(\omega) I_{mn} e^{i\omega t}. \tag{14}$$

By substituting Eq. (14) into Eq. (9), one can find the frequency response function of the plate’s deflection \(H_w(x, y, \omega)\),

$$H_w(x, y, \omega) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \psi_{mn}(x) \phi_{mn}(y) H_{mn}(\omega) I_{mn}. \tag{15}$$

Consequently, the frequency response function of the plate’s absolute displacement \(H_{w_a}(x, y, \omega)\) is

$$H_{w_a}(x, y, \omega) = 1 + H_w(x, y, \omega). \tag{16}$$

In view of the frequency response functions and the power spectral density of the input, the variances of the plate’s deflection and acceleration at different points can be calculated. The input is considered ergodic and stationary with one-sided displacement PSD of \(S_{w_a}(\omega)\) (or acceleration PSD of \(S_{w_a}(\omega)\)) and zero mean value. Hence, the mean square deflection of the plate at point \((x, y)\) can be calculated as [11]

$$E\left[\frac{\dot{w}^2(x, y, t)}{2}\right] = \int_0^\infty \left| H_w(x, y, \omega) \right|^2 S_{w_a}(\omega) \, d\omega \tag{17}$$

Similarly the mean square of the plate absolute acceleration at this point is given by

$$E\left[\frac{\ddot{w}^2(x, y, t)}{2}\right] = \int_0^\infty \left| \frac{\partial^2 H_w(x, y, \omega)}{\partial \omega^2} \right|^2 S_{w_a}(\omega) \, d\omega \tag{18}$$

In the above equations, \(\psi_{mn}(x, y)\) is the undamped normal mode shape or eigenfunction of the \(mn\)th mode of the plate. The mode shapes depend on the system and its boundary condition. The exact closed-form mode shapes for the four-edge clamped (CCCC) boundary condition of a thin rectangular plate are given as [9,19]

$$\psi(x, y) = \left( -\cos \alpha_1 x + \frac{\alpha_2}{\alpha_1} k_1 \sin \alpha_1 x + \cosh \alpha_2 x - k_1 \sinh \alpha_2 x \right)$$

$$\cdot \left( -\cos \beta_1 y + \frac{\beta_2}{\beta_1} k_2 \sin \beta_1 y + \cosh \beta_2 y - k_2 \sinh \beta_2 y \right) \tag{19}$$

where \(k_1\) and \(k_2\) are defined as

$$k_1 = \frac{\cos \alpha_1 a - \cos \alpha_2 a}{\alpha_1 \sin \alpha_1 a - \sinh \alpha_2 a}, \tag{20}$$

and,
The attenuation factor of a clamped plate is quite similar in impingement and nozzle plates are quite close, the surface plot attenuation factor. Since the first few natural frequencies of the plate model employed here.

\[
k_2 = \frac{\cos \beta_1 b - \cosh \beta_2 b}{\sin \beta_1 b - \sinh \beta_2 b}
\]

Furthermore, the eigenvalue equations of a clamped plate are as follows \[9,19\],

\[
\begin{align*}
1 - \cos \alpha_1 a \cos \alpha_2 a & = \alpha_1^2 - \alpha_2^2 \\
\sin \alpha_1 a \sin \alpha_2 a & = 2 \alpha_1 \alpha_2 \\
1 - \cos \beta_1 b \cos \beta_2 b & = \beta_1^2 - \beta_2^2 \\
\sin \beta_1 b \sinh \beta_2 b & = 2 \beta_1 \beta_2
\end{align*}
\]

In Eqs. \((19)\) to \((22)\), \(\alpha_1, \alpha_2, \beta_1\) and \(\beta_2\) are mode dependent unknowns. These unknowns along with the undamped natural frequency of the plate can be determined by solving a set of five nonlinear equations. Two out of the five equations are the eigenvalue equations given by Eq. \((22)\) and the other three equations are as follows,

\[
\begin{align*}
\beta_1^2 + \beta_2^2 & = 2k^2 \\
\alpha_1 & = \sqrt{k^2 - \beta_1^2} \\
\alpha_2 & = \sqrt{k^2 + \beta_1^2}
\end{align*}
\]

where,

\[
k^2 = \omega_{\text{max}} \sqrt{\rho / D}.
\]

For information on how to solve these five nonlinear equations please refer to Refs. \[9,20\]. The reader may refer to Ref. \[9\] for the validity of the plate model employed here.

3. Results and discussion

The random vibration excitation considered in this article is an ergodic and stationary random vibration with PSD profile shown in Fig. 9. This figure is obtained experimentally in a particular application. The vertical axis of the figure is in decibels and in unit of \(g^2 / \text{Hz}\). The flat part of the profile corresponds to PSD level of 0.50 \(g^2 / \text{Hz}\).

Fig. 9. PSD profile of the base random acceleration.

The optimum damping ratio varies from 0.13 to 0.21 for isolation factor using the proposed model is 0.13 resulting in an attenuation factor of 139.4 at the nozzle plate, while this value is equal to 0.17 using the traditional SDOF model which results in attenuation factor of 23.9. That means simplifying the mechanical model from continuous to SDOF introduces 31% error in the optimum damping ratio and 83% error in the corresponding attenuation factor. The optimum damping ratio varies from 0.13 to 0.21 for isolation frequency range of \([20, 50] \text{ Hz}\) using the proposed model while it varies from 0.17 to 0.31 for the same frequency range if the traditional model is used.

Another parameter that directly affects the thermal performance of the jet impingement cooling system is impingement height. Impingement height refers to the gap between the nozzle and the impingement plates. From a thermal point of view, this parameter has an optimum value, and hence minimum deviation in the impingement height is desired. Unlike the traditional model, the new model provides the opportunity to calculate the change in the impingement height due to the transmitted vibration to the plates. Reduction in the impingement height change at the center of the plates as compared to a hard-mount case is illustrated in Fig. 12 as a function of the isolation system parameters. 2D cuts of this surface are plotted in Fig. 13. According to the last two figures, there is an optimum damping ratio maximizing the reduction in impingement height change. This optimum value varies from 0.33 to 0.44 for isolation frequency range of \([20, 50] \text{ Hz}\).

It should be noted that since the center point of a clamped rectangular plate is on the nodal line of its second mode, the response of the plates at their centers in this case study is mainly dependent on the plates' first modes; however, if the response at other points of the plates that do not lie on the nodal line is of interest, or if the plates are thinner or made of materials with smaller trend and numerical values to that of the nozzle plate, and hence it is not presented here.

Fig. 11 shows dependency of the attenuation factor at the center of the nozzle and impingement plates on isolation damping ratio for different isolation frequencies in a 2D graph as compared to that of the traditional SDOF model. This figure shows existence of an optimal damping ratio better than Fig. 10 does. According to Fig. 11, optimum damping for maximizing attenuation factor at both nozzle and impingement plates is the same. Fig. 11 also shows how much different the results of the proposed model are from those of the traditional SDOF model. According to Fig. 11 not only are the attenuation factors much different between these two models but the optimum damping values are also different. For instance at a given isolation frequency of 20 Hz, optimum damping ratio using the proposed model is 0.13 resulting in a attenuation factor of 139.4 at the nozzle plate, while this value is equal to 0.17 using the traditional SDOF model which results in attenuation factor of 23.9. That means simplifying the mechanical model from continuous to SDOF introduces 31% error in the optimum damping ratio and 83% error in the corresponding attenuation factor. The optimum damping ratio varies from 0.13 to 0.21 for isolation frequency range of \([20, 50] \text{ Hz}\) using the proposed model while it varies from 0.17 to 0.31 for the same frequency range if the traditional model is used.
flexural rigidity so that higher number of modes are excited (or the excitation frequency range is wider), the efficacy of the generic formulation and methods presented in this article will be even more highlighted.

Due to stringent rattle space requirements, nowadays compactness is no longer an option but a must. Thus, in addition to vibration of critical components of the system, vibration travel of the whole enclosure or chassis should also be taken into account. Fig. 14 depicts vibration travel of the chassis as a function of the isolation system properties. According to the figure, for a wide range of isolation frequency and damping, vibration travel decreases as the isolation frequency or damping increases.

It was shown that the new proposed model provides significantly more accurate results of the dynamic response of the system than the traditional model; hence, it is more appropriate for optimal design of the vibration isolation system. It also provides some
To design an optimum isolator for a jet impingement cooling system, first it should be determined whether the thermal performance and reliability of the cooling system is more sensitive to the transmitted acceleration to the plates or to the change in the impingement height (the optimum damping ratio at a given isolation frequency is different for either of these criteria). For instance, for the case study of the impingement cooling system presented in this article, the system is more sensitive to the transmitted acceleration than to the change in the impingement height. This is because the change in the impingement height is very small due to the thickness of the plates. After either of the two criteria is selected, an isolation frequency and its corresponding damping ratio should be chosen for maximizing the attenuation factor or minimizing the change in the impingement height (depending on the chosen criterion). When selecting the isolation frequency and its corresponding optimum damping ratio, the smallest possible isolation frequency should be chosen with the condition that the associated vibration travel does not exceed the allowed rattle space.

4. Conclusion

In this article, for a more efficient vibration isolation system design, a new mathematical model is proposed for a double-chamber submerged jet impingement cooling system mounted in a chassis. Every submerged jet impingement cooling system normally consists of nozzle and impingement plates. Unlike the traditional vibration isolation system models, the new model presented in this article takes the critical flexible components of the cooling system, namely the impingement and nozzle plates, into account. The plates are integrated to the model of the rest of the system as thin plates clamped at their edges, and response of the system to random base vibration is then mathematically formulated. Next, the vibration isolation system properties are optimized for the response of the critical components rather than for the whole chassis (the traditional method) subject to an allowed rattle space. The modeling and methodology presented in this paper are very generic and could be applied to a wide range of jet impingement cooling systems. The methods presented will be even more efficient and different from those of an SDOF model if the nozzle and impingement plates are more flexible with more number of responsive modes.

The results show that there is an optimum damping maximizing the attenuation factor at the nozzle and impingement plates. The optimum damping values for the proposed and traditional models are different. Furthermore, it was shown that the simplifying assumptions of the traditional SDOF model affect the attenuation factors significantly. It was also shown that there is an optimum damping for a given isolation frequency that minimizes the change in impingement height. This optimum damping differ from the one maximizing the attenuation factor. Finally, it was shown
that for a wide range of isolation properties, vibration travel of the chassis increases as the isolation frequency or damping decreases. The vibration travel constraint limits the minimum possible value chosen for the isolation frequency.

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References


